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Communication

Rapid surface-to-volume ratio and tortuosity measurement using Difftrain

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Abstract

Analysis of diffusion measurements as a function of observation time (Δ), to calculate surface-to-volume ratios (S/V) and tortuosities (κ), is a useful tool in the characterisation of porous media using NMR. However, using conventional pulsed field gradient (PFG) measurements, this requires long total experiment times (typically hours). Here, we show how the rapid diffusion measurement pulse sequence, Difftrain, can be used to provide the required experimental data much more rapidly (typically within minutes) with a consequential reduction in total experiment time of typically over an order of magnitude. Several novel modifications to the Difftrain pulse sequence are also presented to tailor it to this particular application; these include a variable delay between echoes (to ensure optimal echo position with respect to Δ) and a variable tip angle for the refocusing pulse (to ensure optimal use of available signal). Difftrain is applied to measure both S/V and κ for a model glass bead pack; excellent agreement is found with both a conventional PFG measurement and with a bulk gravimetric measurement of S/V.

Keywords: NMR; Difftrain; Porous media; Surface-to-volume; Tortuosity

1. Introduction

Measurement of diffusion using NMR pulsed field gradient (PFG) techniques has a wide variety of applications. In particular, measurements of apparent diffusion for a fluid whose diffusion is restricted by a porous structure can provide information regarding the characteristic length scales of that structure. However, it is generally a time consuming procedure, mainly due to the need for the magnetisation to relax between each signal excitation, which typically limits conventional PFG techniques to systems at equilibrium.

A large variety of NMR pulse sequences and methods are available to allow rapid measurement of diffusion. The simplest of these use conventional PFG measurements, but limit the collection of signal attenuation data to just two points [1], where the error is minimised through careful selection of the second gradient strength. A similar approach has been applied to the determination of the surface-to-volume ratio (S/V) for a porous silica gel [2]. As the observation time (Δ) is increased, the gradient strength, g, is reduced to keep $q^2\Delta$ (where $q = \gamma g \delta$) constant. This means that any variation in the signal intensity with observation time (ignoring T_1 relaxation) is due to restricted diffusion in the system; only one experiment is thus theoretically needed per Δ required. An alternative approach is to minimise the repetition of experiments needed for phase cycling through use of unbalanced field gradients [3]. It is also possible to use a varying gradient strength over the sample in order to acquire information for multiple gradient strengths during a single excitation either through use of the z^2 shim [4] or else by using a frequency swept 180° pulse applied in conjunction with the encoding

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gradient [5]. Spin echoes have also been used to measure time dependent diffusion in porous media [6], but suffer from poor signal to noise at large Δ due to low T_2 values. Other alternatives include the BURST technique [7] which uses multiple small flip angle pulses in the presence of a static field gradient to develop several coherence pathways (each equivalent to a different gradient strength) that are refocused using a spin echo. This technique can be extended [8] through use of a stimulated echo that allows a portion of the magnetisation to be refocused, with the multiple coherence pathways resulting in a train of echoes shortly afterwards. A further modification is multiple modulation multiple echoes (MMME) [9] which is not limited to small flip angles and does not eliminate most coherence pathways resulting in a far greater number of echoes. The drawback of these types of pulse sequences, with respect to the type of application being considered here, is that it is difficult to associate sequential echoes (with varying q) to a single Δ unless the time spent in storage is much greater than the encoding time.

In this work, the fast diffusion measurement pulse sequence Difftrain [10] is used to acquire the necessary signal attenuation data for diffusion measurement in a significantly reduced total acquisition time. In contrast to the BURST type pulse sequences mentioned previously, Difftrain does have a unique Δ associated with each echo acquisition regardless of gradient strength (which is required in this study). Instead of holding the observation time constant and then varying the gradient strength, g, between experiments (as in conventional PFG), Difftrain varies Δ for a set value of g within an experiment, acquiring a train of echoes at increasing Δ using a partial excitation of the stored encoded magnetisation. With appropriate selection of a set of g values, this enables a rapid determination of the apparent diffusion coefficient, D_{app}, as a function of Δ, as required for the determination of the surface-to-volume ratio (S/V) and tortuosity (κ) characterising the porous medium in which constrained diffusion is occurring. In addition, Difftrain makes use of a stimulated echo and alternating pulsed gradients which minimise the influence of background magnetic gradients [11], both of which are generally required for quantitative application of NMR diffusion measurement in porous media. Difftrain has been used to rapidly size droplets in emulsions [12] and measure displacement propagators for flow through a packed bed [13]. Here, we present further modifications of the Difftrain pulse sequence that reduce the total acquisition time by truncating echoes, has a variable delay between echo acquisitions and has a variable recovery pulse flip angle. The modified Difftrain sequence is used to acquire NMR signal attenuation data for a water saturated glass bead pack and hence to find S/V and κ for the confining pore space. The results are then compared to conventional PFG measurements, literature values and a bulk gravimetric measurement of S/V.

2. Background

At comparatively short Δ, the majority of fluid molecules held within a saturated porous medium will experience free diffusion. Only molecules in a thin layer adjoining the solid surface can possibly be subject to any restriction of their self-diffusion. The thickness of this layer is related to the diffusion length, $\sqrt{2D_0\Delta}$. It has been shown that at short Δ the apparent diffusion coefficient, D_{app}, in a porous medium with smooth boundaries is given by [14]:

$$\frac{D_{app}}{D_0} \approx 1 - \frac{4}{9\sqrt{\pi}}\frac{S}{V}\sqrt{D_0\Delta'}, \tag{1}$$

where S/V is the pore surface-to-volume ratio and D_0 is the free self-diffusion coefficient of the confined fluid. Δ' is a modified observation time (defined in [15]) for use in Eq. (1), required at comparatively short observation times due to the effects of diffusion during gradient pulses of finite width.

Fig. 1 shows a schematic of D_{app}/D_0 against $\Delta^{0.5}$ for a typical porous medium. S/V can be found from the gradient at short Δ using Eq. (1). At long Δ the diffusing molecules explore the connectivity of the porous medium and the ratio of the apparent diffusion coefficient to the free diffusion coefficient approaches an asymptote equal to the inverse of tortuosity, $\frac{1}{\kappa}$, [16]:

$$\lim_{t\to\infty}\left[\frac{D_{app}}{D_0}\right] = \frac{1}{\kappa}. \tag{2}$$

3. Experimental

3.1. Difftrain pulse sequence developments

The standard 13 interval alternating pulsed gradient stimulated echo (APGSTE) pulse sequence [11] used in this

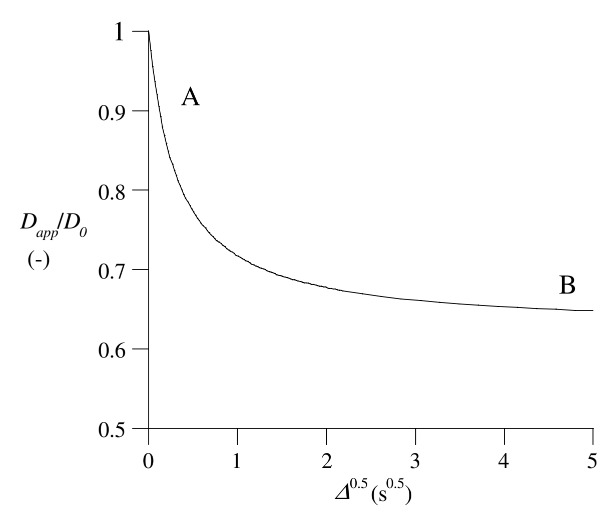

Fig. 1. A schematic typical graph of D_{app}/D_0 vs. $\Delta^{0.5}$ for a porous medium. The gradient in region A provides information about S/V (Eq. (1)) and the asymptotic value reached in region B is the inverse of κ (Eq. (2)).

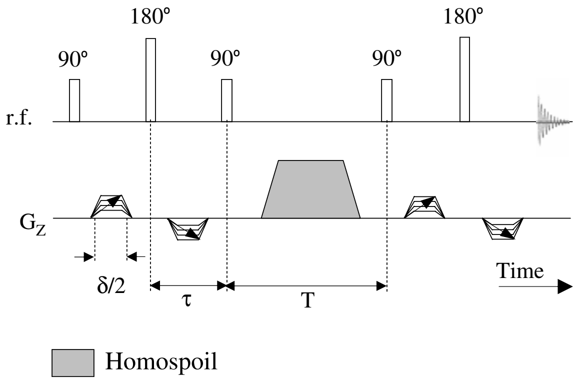

Fig. 2. Alternating pulsed gradient stimulated echo (APGSTE) sequence showing gradient duration δ, inter-pulse encoding time τ and evolution time T.

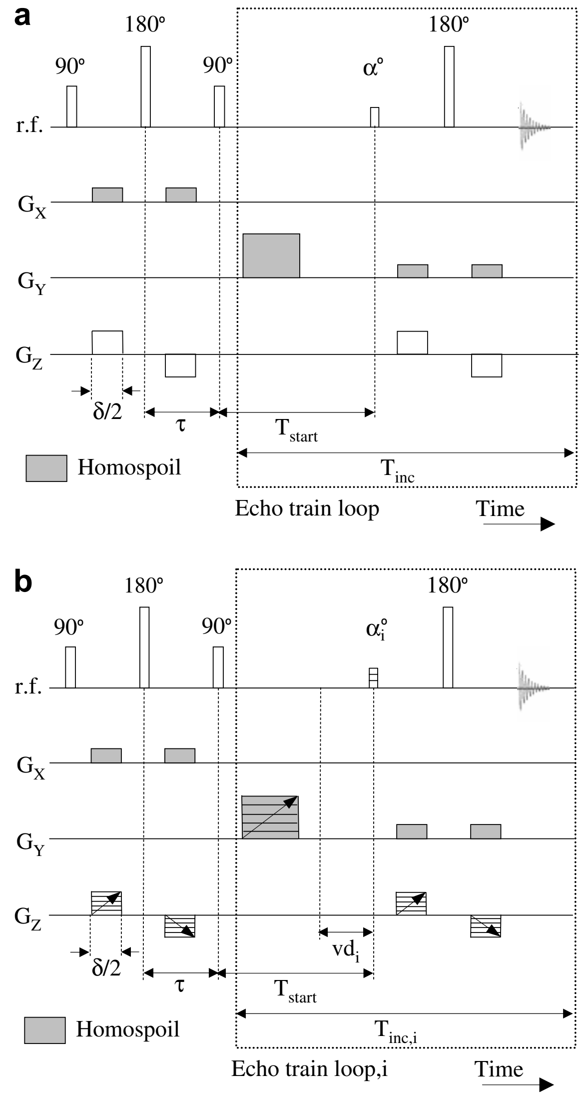

Fig. 3. (a) Original and (b) modified Difftrain pulse sequences showing gradient duration δ, inter-pulse encoding time τ, initial evolution time T_{start}, incremental evolution time (between subsequent pulses) T_{inc}, variable flip angle α and variable delay time vd. T_{inc}, α and vd are a function of echo train loop number in (b).

work is shown in Fig. 2. The magnetisation is excited, spatially encoded and then effectively returned to storage (where T_1 relaxation occurs) during the first three r.f. (radiofrequency) pulses and accompanying gradients. Bipolar gradients (of length δ) are preferred to reduce the effect of background magnetic field gradients. After the evolution time period, T, the magnetisation is rotated back into the transverse plane, and decoded before an FID is acquired. The experiment is repeated at various values of g, δ or Δ. An apparent diffusion coefficient, D_{app}, is calculated from the ratio of signal intensity in the presence of the magnetic pulsed field gradients, I, and the signal intensity in the absence of magnetic pulsed field gradients, I_0, using the Stejskal–Tanner equation at low attenuation levels [11]:

$$\ln\left(\frac{I}{I_0}\right) = -D_{\text{app}}\gamma^2 g^2 \delta^2 \Delta, \tag{3}$$

where γ is the gyro-magnetic ratio and Δ is defined as

$$\Delta = T + \frac{3\tau}{2} - \frac{\delta}{3}, \tag{4}$$

where T, τ and δ are all defined in Fig. 2.

The Difftrain pulse sequence (Fig. 3a) is also based on the standard 13 interval APGSTE sequence and thus the magnetisation is initially encoded as before but echoes are acquired at increasing Δ using a series of stimulated echoes, requiring that for each echo only a portion of the signal is rotated down onto the transverse plane and subsequently used and lost. The magnitude of the signal used for each echo is determined by the flip angle, α; $\sin\alpha$ is rotated from the stored magnetisation to form the echo, leaving $\cos\alpha$ in storage for future echoes. An apparent diffusion coefficient can be extracted from the resultant train of echoes at increasing Δ using Eq. (3) and (4) where T_i is defined by

$$T_i = T_{\text{start}} + (i-1)T_{\text{inc}}, \tag{5}$$

and T_{start} and T_{inc} are defined in Fig. 3(a); i is the echo number. Due to T_1 relaxation and the diminishing pool of magnetisation, it is necessary to correct the magnitude of the echoes in the train; a second echo train is thus acquired in the absence of encoding gradients. The attenuation caused purely by diffusion can then be isolated by dividing the first echo train by the second.

Fig. 3(b) shows the modified Difftrain pulse sequence used in this work. For a typical system under investigation using a standard Difftrain pulse sequence, acquiring a complete FID takes typically tens of milliseconds. In order to probe the short observation times necessary to determine S/V for the pore space in our application, this must be reduced significantly and therefore only the initial portion (64 complex points) of the FID was acquired. This reduces the ability to chemically resolve a spectrum from the sample; this is however unnecessary for the current application. The remaining magnetisation in the transverse plane is removed using a homospoil before subsequent α partial excitation pulses. The number of echoes that may be acquired in the train is limited by the available encoded magnetisation. For each echo, a portion of the magnetisation is recovered from storage and is subsequently necessarily lost. Therefore during each subsequent echo, a portion of a diminishing reservoir of encoded magnetisation is being excited and, exacerbated by T_1 relaxation, this limits the number of echoes and the maximum final observation times achievable for a given signal to noise (S/N) ratio. In order to reduce this effect, the power provided to the α pulse was allowed to vary during the train of n echoes resulting in a varying flip angle, $\alpha_{i\ (i=1\ldots n)}$. Ideally the S/N ratio should be equal in all acquired echoes for $g = 0$ G/cm; in order to achieve this $\alpha_{i\ldots,n}$ must satisfy (ignoring comparatively short periods of T_2 relaxation between storage):

$$e^{\frac{\Delta_1}{T_1}}\cos\alpha_1 = e^{\frac{\Delta_2}{T_2}}\cos\alpha_2\sin\alpha_1 = \cdots = e^{\frac{\Delta_n}{T_1}}\cos\alpha_n\prod_1^{n-1}\sin\alpha_i, \quad (6)$$

where the subscripts refer to echo number in the train of n echoes. There are an infinite number of solutions to Eq. (6). If we choose to use all available signal, then $\alpha_n = 90°$ and a unique solution exists. In principle, implementation of Eq. (6) should enable a single shot application of Difftrain (if phase cycling is neglected). In practice, the effect of manipulating α for the initial echoes in the train is minimal as S/N is usually more than sufficient.

In order to spread the echoes appropriately a variable delay between echoes was also introduced. For the current application, the first five echoes were collected in rapid succession in order to have sufficient resolution at short observation times to determine S/V. Three echoes were then acquired at much longer observation times to collect data on the asymptotic D_{app} to measure κ. These last three echoes were more widely spread in order to check that D_{app} was no longer a function of observation time and was therefore at its asymptotic value. The observation time associated with each echo can be found from:

$$T_i = T_{start} + \sum_1^{i-1} T_{inc,i}, \quad (7)$$

followed by application of Eq. (4). A further modification necessary to the pulse sequence is a ramped homospoil gradient [17]. Using homospoils between the refocusing pulses is essential to remove unwanted magnetisation, but undesired coherence pathways can be refocused by future homospoils and to prevent this, a sequentially ramped homospoil was implemented in the echo train.

3.2. Experiments conducted

All experiments were performed on a Bruker Biospin 400 wide-bore spectrometer, using a 20 mm diameter birdcage r.f. coil and an orthogonal gradient set with a maximum gradient strength of 150 G/cm. A 12 mm deep bead pack was made from 100 μm glass ballotini (QA Equipment Ltd.) flooded with de-ionised water in a flat-bottomed 16 mm inner diameter glass vial. Excess water was removed from the top of the bead pack using a syringe to eliminate signal from this free-diffusing water.

Table 1
List of the flip angles used in series A, B and C

Echo	Series A (°)	Series B (°)	Series C (°)
1–5	5.6	5.6	5.6
6	5.6	11.3	12
7	5.6	11.3	15
8	5.6	11.3	19

The modified Difftrain pulse sequence was implemented to find D_{app} for the fluid in the bead pack as a function of Δ. The actual observation times and gradient strengths used must be tailored to the particular system under investigation. In order to determine both S/V and κ, the apparent diffusion coefficient has to be determined at both long and short observation times; five points were collected as rapidly as possible at short observation times (5, 17, 28, 40 and 52 ms) and three (more widely spaced) points were collected at considerably longer observation times (2065, 2575 and 3085 ms). Three different series of flip angles were used (Table 1). Series A has constant flip angles, series B has increased flip angles for the last three echoes and Series C has been designed, with Eq. (6) in mind, to spread the available signal more evenly between echoes. Three different values of g were used in turn: one common low value (1 G/cm), a moderate value (2.5 G/cm) for measurement of κ and a comparatively larger value (12 G/cm) for measurement of the S/V ratio; in all cases $\delta = 2$ ms. In order to achieve appropriate levels of attenuation, a higher gradient strength is needed for the short Δ measurements compared to the long Δ measurements. All homospoil gradients were applied for durations of 2 ms; the constant homospoils were applied at a strength of 1.46 G/cm and the changing homospoil was ramped linearly from 4.38 to 81.03 G/cm.

The digitised point of greatest magnitude in the acquired FID for each observation time was extracted and used to determine D_{app} using the two appropriate gradient strengths and Eq. (3). Eq. (1) was applied to the first five D_{app} values, as a function of Δ, and used to determine S/V and D_0. The last three values of D_{app} were inspected to ensure that the asymptotic limit had been reached and hence κ was determined using Eq. (2). In the current application, it was not necessary to correct with a reference echo train ($g = 0$), as each echo train (using different gradient strengths) was otherwise acquired using the same experimental conditions.

4. Results and discussion

Fig. 4 compares the absolute signal intensity, I, for the train of eight echoes as a function of gradient strength used. This is shown for all three series of flip angles used (series A–C in Table 1). What is immediately obvious is

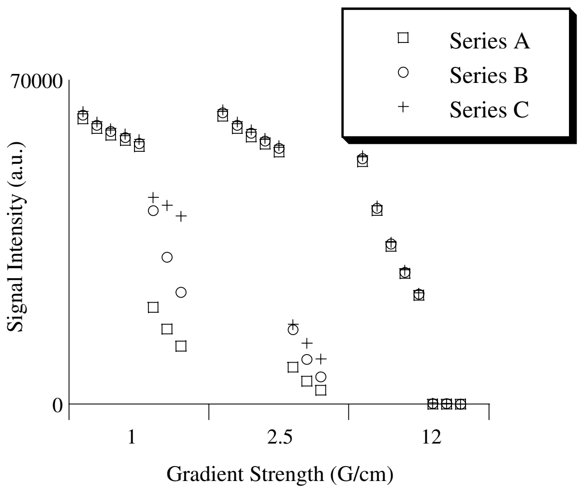

Fig. 4. Points of maximum signal intensity extracted from each FID for the three series given in Table 1. Three different values of g were acquired (1, 2.5 and 12 G/cm) and each gradient strength consists of a train of eight echoes at increasing Δ (5, 17, 28, 40, 52, 2065, 2575 and 3085 ms).

the minimal attenuation for the first five echoes in the trains between gradient strengths of 1 and 2.5 G/cm, necessitating the use of a gradient strength of 12 G/cm for the determination of D_{app} for these comparatively low values of Δ. For the last three echoes, the attenuation for this stronger gradient is clearly excessive, necessitating the use of the lowest two gradient strengths to determine D_{app} for these longer values of Δ. With respect to the three series of flip angles, it can be seen that there is an improvement in signal intensity from series A to series B to series C for the longer observation times. All of these experiments were carried out using the same experimental conditions (with the exception of the flip angle variation) and so the S/N ratio is proportional to the absolute signal amplitude shown in Fig. 4. Clearly if the choice of flip angles results in a more even un-attenuated distribution of I, better quality data, characterised by an improved S/N ratio, is produced.

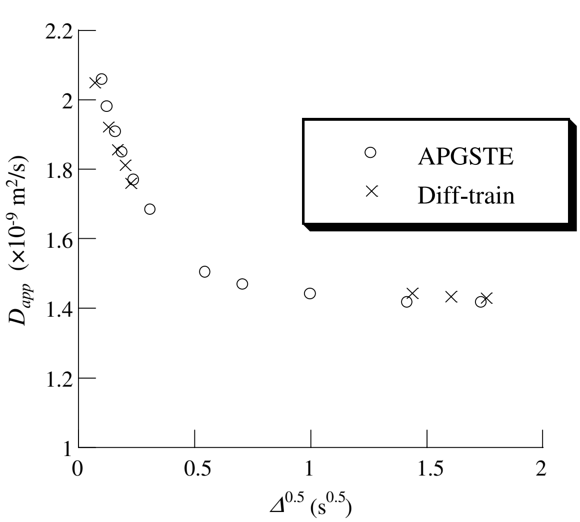

Fig. 5. D_{app} versus $\Delta^{0.5}$ as measured using both APGSTE and Difftrain.

Table 2
Comparison of S/V, 'pore size' (assuming spherical pores) and κ as found using APGSTE, gravimetric methods and Difftrain

	APGSTE	Theory	Difftrain (12 G/cm)	Difftrain (17 G/cm)
S/V (m^{-1})	99,700	98,730	100,000	107,000
'Pore size' (μm)	32.0	30.4	29.9	26.4
κ (−)	1.73	1.63	1.66	1.71

A comparison of D_{app} plotted versus $\Delta^{0.5}$ for both APGSTE and Difftrain (using flip angle series C) is shown in Fig. 5. The APGSTE method has also been used to measure D_{app} at intermediate times. It can be seen that the agreement between the two methods is excellent. At long Δ, it can be seen that D_{app} is no longer a function of Δ. The resultant values of S/V and κ are presented in Table 2. The packed bed was dried and the porosity (ϕ) calculated to be 37.8%, which is in good agreement with literature values for randomly packed monodisperse spherical bead packs [18] and from this the theoretical S/V value [16] and approximate value of κ [19] can be calculated using:

$$\frac{S}{V} = \frac{6}{\phi d_p}(1-\phi) \tag{8}$$

and

$$\frac{D_{app}}{D_0} = \frac{1}{\kappa} \approx \frac{1}{\sqrt{\phi}}. \tag{9}$$

where d_p is bead diameter. The agreement between the Difftrain data, the APGSTE data and these predictions, as presented in Table 2, is excellent.

It is desirable to have a significant signal attenuation to improve the quality of the diffusion coefficient fit, but care must be taken not to violate the assumption of a Gaussian distribution of phase required by Eq. (3), [20]. The experiment was repeated using 17 G/cm for the highest gradient strength and D_{app} was found to be lower than expected. The calculated values of S/V and κ are also shown in Table 2 and are clearly in poorer agreement with the expected values. The total acquisition time for the whole data set using Difftrain is 2.5 min. compared with 75 min. when using APGSTE. If just the value of S/V or κ is desired, then total acquisition time would be 1.66 min.

5. Conclusion

The Difftrain pulse sequence has been modified to make it suitable for the rapid acquisition of diffusion data to calculate S/V and κ for a porous medium. It has been changed to allow variation in the delay time between each of the acquisitions in the echo train, acquire a fraction of the complete FID, destroy unwanted magnetisation using a variable homospoil and increase the flip angle of the refocusing pulses for later echoes in the train to improve S/N. These modifications to the Difftrain pulse sequence allow S/V and κ values for a 100 μm glass bead-pack to be measured in 2.5 min. The results are in excellent agreement with both the gravimetrically-determined value and with a similar experiment carried out using the conventional APGSTE sequence. Future work will focus on further optimisation of flip angles based on Eq. (6) and the application of the technique to dynamic systems, in particular drying porous samples.

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